RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, MARCH 2021

THIRD YEAR [BATCH 2018-21]

Date : 16/03/2021 **PHYSICS (HONOURS)**

Time : 11 am - 1 pm **Paper : V [Gr. B]** Full Marks : 50

Answer **any five** questions [5 × 10]

(Answer at least two questions from each unit and one question from any unit)

Unit - I

1. a) Give a set of generalised coordinates needed to completely specify the motion of (i) a particle constrained to move on the surface of a sphere and (ii) a circular disk rolling on a horizontal plane.[0.5×2]

   b) Classify each of the following according as they are scleronomic or rhenomic, holonomic or nonholonomic, conservative or nonconservative and unilateral or bilateral: (i) motion of a car on the surface of Earth and (ii) a cylinder rolling down an inclined plane. [2+2]

   c) Two particles of masses m_1 and m_2 are located on a frictionless double incline and connected by an inextensible massless string passing over a smooth peg. (i) Use the principle of virtual work to show that for equilibrium, we must have $\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{m_2}{m_2}$, where α_1 and α_2 are the angles of the incline. (ii) Apply D'Alambert's principles to describe the motion of the particles. [2+3]

2. a) A particle of mass m is projected with initial velocity u at an angle α with the horizontal. Write down the Lagrange's equation of motion. Which quantities are conserved? [2+1]

   b) (i) The Lagrangian for a system of one degree of freedom can be written as,

   $$L = \frac{m}{2}[\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2\omega^2]$$

   What is the corresponding Hamiltonian? Is it conserved? [2+1]

   (ii) Introduce a new coordinate defined by $Q = q \sin \omega t$. Find the Lagrangian in terms of the new coordinate and the corresponding Hamiltonian. Is it conserved now? [1+2+1]

3. a) Prove that the shortest distance between the points on the surface of a sphere is the arc of the great circle connecting them. [Hint: The line element on a sphere of radius a is $ds^2 = a^2(d\theta^2 + \sin^2 d\varphi^2)$.] [5]

   b) A Hamiltonian of one degree of freedom has the form

   $$H = \frac{p^2}{2\alpha} - 2qpe^{-\alpha t} + \frac{ba}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2}$$

   where b, a, α and k are constants. (i) Find the Lagrangian corresponding to this Hamiltonian.

ii) Find an equivalent Lagrangian that is not explicitly dependent on time. (iii) What is the Hamiltonian corresponding to this second Lagrangian, and what is the relationship between the two Hamiltonians? [2+1+1+1]

4. a) The potential for an anharmonic oscillator is $U = \frac{kx^2}{2} + \frac{bx^4}{4}$ where k and b are constants. Find the Hamilton's equations of motion. [2]

b) A system of two degrees of freedom is described by the Hamiltonian $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$. Show that $F_1 = \frac{p_1 - aq_1}{q_2}$ and $F_2 = q_1q_2$ are constants of motion. Are there any other independent algebraic constants of the motion? Can any be constructed from Jacobi's identity?[2+2+2+2]

5. Three simple pendulum of same mass m and length l are hanging from the ceiling and the masses are connected by two springs of same spring constant k. The system have potential energy given by, $V = \frac{1}{2}[(mgl + kl^2)(\theta_1^2 + \theta_2^2 + \theta_3^2) - 2kl(\theta_1\theta_2 + \theta_2\theta_3 + \theta_1\theta_3)]$. Find the normal frequencies and normal modes. Also show how generalised coordinates are related to the normal coordinates. [10]

Unit - II

6. a) Find the square roots of -15 - 8i.

b) Given $\lim_{z \to z_0} g(z) = B$, prove that $\lim_{z \to z_0} \frac{1}{g(z)} = \frac{1}{B}$ if B≠0, where z is a complex variable.

c) Discuss the branch points of $\log(z^2-1)$. [3+4+3]

7. a) Locate and name the singularities in the finite z plane for f(z) = sec(1/z).

b) Evaluate using Cauchy's theorem $\oint_C \frac{dz}{z-a}$ where C is any simple closed curve C and z=a is (i) outside C, (ii) inside C.

c) Find the residues of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. [4+4+2]

8. a) Suppose f(z) is analytic in a region R. Prove that f'(z), f''(z), . . . are analytic in R. [3]

b) Let f(z)= ln(1 + z), where we consider the branch that has the zero value when z=0.

(i) Expand f (z) in a Taylor series about z = 0.

(ii) Determine the region of convergence for the series in (a). [3+1]

c) Show that $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)^2(x^2+2x+2)} = \frac{7\pi}{50}$ [3]

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